Řešení - IX.A - 9.6. 2022

1) a) $x^2-10x = (x-10)(x+10)$

b) $a^2-2ab = a(a-2b)$

c) $20a^2-20a+5 = 5(4a^2-4a+1) = 5(2a-1)^2$

d) $3x^2-3 = 3(x^2-1) = 3(x-1)(x+1)$

2) $\frac{12x^2-3}{12x^2+6x} = \frac{3(4x^2-1)}{6x(2x+1)} = \frac{3(2x-1)(2x+1)}{6x(2x+1)} = \boxed{\frac{2x-1}{2x}}$

$x \neq 0$

$x \neq -\frac{1}{2}$

3) a) $\frac{ab^2-3ab}{3ab^2-6b^2} \cdot \frac{6ab-12b}{2ab-6a} = \frac{ab(b-3)}{3b^2(a-2)} \cdot \frac{6b(a-2)}{2a(b-3)} = \underline{1}$

$a \neq 0$

$b \neq 0$

$a \neq 2$

$b \neq 3$

b) $\left(\frac{a}{a-b}+1\right) \cdot \frac{6a^2-12ab+6b^2}{12a^3-3ab^2} = \frac{a+a-b}{a-b} \cdot \frac{6(a-b)^2}{3a(4a^2-b^2)} =$

$= \frac{2a-b}{a-b} \cdot \frac{6(a-b)(a-b)}{3a(2a-b)(2a+b)} = \boxed{\frac{2(a-b)}{a(2a+b)}}$

$a \neq b$ $\quad a \neq \frac{b}{2}$

$a \neq 0$ $\quad a \neq -\frac{b}{2}$

c) $\frac{12a^4b}{9a^2b} \cdot \frac{6ab^3}{8a^3b^2} = \frac{12 \cdot 6 \cdot a^5 b^4}{9 \cdot 8 \cdot a^5 b^3} = \boxed{b}$

$a \neq 0$

$b \neq 0$

4) a) $\dfrac{\frac{8x^2-32}{2-2x}}{\frac{3x-6}{x^2-x}} = \dfrac{\frac{8(x^2-4)}{2(1-x)}}{\frac{3(x-2)}{x(x-1)}} = \dfrac{4(x-2)(x+2)\cdot x\cdot(x-1)}{(1-x)\cdot 3\,(x-2)} = \dfrac{4x(x-2)}{3}$

$-(x-1)$

$x \neq 1$
$x \neq 0$
$x \neq 2$

b) $\dfrac{\frac{12a^4b}{9a^2b}}{\frac{6ab^3}{8a^3b^2}} = \dfrac{12a^4b\,8a^3b^2}{9a^2b\,6ab^3} = \dfrac{12\cdot 8\cdot a^4b^5}{9\cdot 6\cdot a^3b^4} = \dfrac{16a^4}{9b}$

$a \neq 0$
$b \neq 0$

5) a) $\dfrac{5}{a^2-8ab+16b^2} + \dfrac{2a}{4b-a} = \dfrac{5-2a(a-4b)}{(a-4b)^2} = \dfrac{5-2a^2+8ab}{(a-4b)^2}$

$(a-4b)^2$; $-(a-4b)$

$a-4b \neq 0$
$\boxed{a \neq 4b}$

b) $\dfrac{x+3}{3} - \dfrac{x^2-3}{x-3} = \dfrac{(x+3)(x-3)-3(x^2-3)}{3(x-3)} = \dfrac{x^2-9-3x^2+9}{3(x-3)} = \boxed{\dfrac{-2x^2}{3(x-3)}}$

$x \neq 3$

c) $\dfrac{x-2}{x+2} - \dfrac{x+2}{x-2} = \dfrac{(x-2)^2-(x+2)^2}{(x+2)(x-2)} = \dfrac{x^2-4x+4-(x^2+4x+4)}{(x+2)(x-2)} = \dfrac{x^2-4x+4-x^2-4x-4}{(x+2)(x-2)} =$

$= \dfrac{-8x}{(x+2)(x-2)}$

$x \neq -2$
$x \neq 2$